

A series of papers by Prof. Alexander I. NAZAROV on the properties of nonlocal operators, in particular, the fractional Laplacians is submitted to the St. Petersburg State University Prize for scientific works. The series contains the following papers:

1. *On fractional Laplacians* // Communications in Partial Differential Equations, V. 39 (2014), N9, 1780-1790. (Q1; IF 1.239)
2. *On the Sobolev and Hardy constants for the fractional Navier Laplacian*, // Nonlinear Analysis – Theory, Methods and Applications, V. 121 (2015), 123-129. (Q1; IF 1.450)
3. *Non-critical dimensions for critical problems involving fractional Laplacians* // Revista Matematica Iberoamericana, V. 32 (2016), N1, 257-266. (Q1; IF 1.039)
4. *On fractional Laplacians – 2* // Annales de l'Institut Henri Poincaré. Analyse Nonlinéaire, V. 33 (2016), N6, 1667-1673. (Q1; IF 2.201)
5. *On fractional Laplacians – 3* // ESAIM: Control, Optimization and Calculus of Variations, V. 22 (2016), N3, 832-841. (Q1; IF 1.295)
6. *Variational inequalities for the spectral fractional Laplacian* // Computational Mathematics and Mathematical Physics, V. 57. 2017. N3. P.373-386 (Q2/Q3; IF 0.774)
7. *Sobolev inequalities for fractional Neumann Laplacians on half spaces* // Advances in Calculus of Variations (2018). 1-19. DOI: 10.1515/acv-2018-0020. (Q1; IF 2.316)
8. *A note on truncations in fractional Sobolev spaces* // Bull. Math. Sci. DOI: 10.1142/S1664360719500012. Published online 28.06.2017. (Q1; IF 1.714)
9. *Fractional Hardy-Sobolev inequalities on half spaces* // Nonlinear Analysis – Theory, Methods and Applications. DOI: 10.1016/j.na.2018.07.002. Published online 30.07.2018. (Q1; IF 1.450)

All papers are joint with Prof. Roberta Musina from Università di Udine, Italy. All papers except for N6 are published in the journals of the first quartile according to the SJR system (Scopus). Article N6 is an invited paper in a special issue of “Journal of Computational Mathematics and Mathematical Physics”, dedicated to the memory of the outstanding Soviet and Russian mathematician S.I. Pohozaev.

The papers are available in the Internet:

1. <https://arxiv.org/abs/1308.3606>
2. <https://arxiv.org/abs/1408.3946>
3. <https://arxiv.org/abs/1311.1788>
4. <https://arxiv.org/abs/1408.3568>
5. <https://arxiv.org/abs/1503.00271>
6. <https://arxiv.org/abs/1603.05730>
7. <https://arxiv.org/abs/1708.01567>
8. <https://arxiv.org/abs/1701.04425>
9. <https://arxiv.org/abs/1707.02710>

Fractional Laplacians and equations with them have been actively studied in last decades throughout the world in various fields of mathematics (Analysis, Partial Differential Equations, Theory of Random Processes) and its applications (Physics, Biology). Hundreds of articles have been written on this topic. Note that the study of such operators and equations is complicated not only by the fact of nonlocality itself, but also by existence of several nonequivalent definitions of fractional Laplacian.

The articles in the series are devoted to the study of the qualitative properties of various fractional Laplacians, their similarities and differences, as well as the problems of solvability and qualitative analysis of solutions of equations with such operators. Fundamentally new results on comparing fractional Laplacians have been obtained, including the practically unexplored case of operators of the order larger than one. For some classes of equations with fractional operators, unimprovable solvability results are established. For analogues of the classical functional Sobolev and Hardy-Sobolev inequalities generated by fractional Laplacians, results on exact constants and their attainability or unattainability are obtained.

The results are achieved by a combination of methods of the theory of Partial Differential Equations, Calculus of Variations, Functional Analysis, Function Theory and Spectral Theory of Operators. A significant part of the approaches to the problems considered is the original development of the authors. In particular, the operator approach was introduced into this field by A.I. Nazarov. He also suggested a new approximation procedure to study the operators of the order larger than one.

The papers of the series were highly appreciated by specialists, and the authors became recognized experts in this field. In particular, A.I. Nazarov in last years has given 6 invited talks on this topic at international conferences.

We also note that the first article in the series is one of the three most read articles published in the “Communications in Partial Differential Equations” for all years. In the Scopus system it has 42 citations.