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Hamza M. Benzerrouk

Modern Approaches in Nonlinear Filtering Theory Applied to Original Problems of Aerospace Integrated Navigation Systems with non-Gaussian noises The series *Saint Petersburg State University Studies in Mathematics* presents final results of research carried out in postgraduate mathematics programs at St. Petersburg State University. Most of this research is here presented after publication in leading scientific journals.

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Printed in Russia by St. Petersburg University Press 11/21 6th Line, St. Petersburg, 199004

ISBN 978-5-288-05426-6 ISSN 2308-3476 © Hamza M. Benzerrouk, 2014

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ABSTRACT

Benzerrouk, Hamza.M.

Modern Approaches in Nonlinear Filtering Applied to Aerospace Integrated Navigation Systems with Non-Gaussian noises Saint Petersburg: Saint Petersburg State University, 2014, 49 p.(+included articles) Saint Petersburg State University Studies in Mathematics, Vol. 4 ISBN 978-5-288-05530-0, ISSN 2308-3476

In integrated navigation systems, the problem of data fusion from multiple sensors is a challenging problem and the methods involved to solve such complex problems are based on Kalman filtering theory and its nonlinear variants algorithms. The challenges in this field are how to obtain the best estimation accuracy and stability of the mathematical methods used in solving state estimation problems, especially in denied measurement environments. Mathematically, this leads with solving the problem of nonlinear estimation in the presence of non-Gaussian noises and then needs new formulations of the previous and modern approaches in nonlinear filtering algorithms, with also the proposal of novel approximations of the lower bounds of estimation. In the linear estimation problems with white Gaussian noises in the system and the measurement, Cramer Rao Lower Bound CRLB is well known to be the optimal lower bound of estimation, however if the noises are correlated or are non-Gaussian, the mathematical problem becomes more complex and all well know techniques and methods in the Gaussian space are transformed into robust or adaptive forms. This dissertation work is devoted to such severe conditions of noises where measurements are assumed affected by non-Gaussian noise, especially Gaussian mixture density.

The second part of this contribution is a practical such as in many applications of autonomous navigation; Unmanned Aerial vehicle UAV or Unmanned Marine vehicle UMV for survey missions, photogrammetric, traffic survey,...etc, the sensors embedded and integrated onboard suffer from intentional and/or unintentional disturbances such as spoofing, jamming, multipath signals which induces a phenomena of measurement outliers. Especially with the use of GNSS signal composed by the previous and modern satellite constellations including "GPS, GLONASS, Galileo and Beidou", and the merging new information technology frequencies used by peoples in urban environment like Smartphones, GSM, WiFi, Bluetooth, onboard the aircrafts, during walking in the city, some interferences problems occur sometimes without any intentional meaning, and cause a major degradation in the navigation process. Interesting and original applications in this dissertation are related to short duration Navigation of UAV, Robot navigation and pedestrian navigation in the city in degraded GNSS environment. Because of the emergence of low cost sensors such as MEMS based sensors including accelerometers, gyroscopes, magnetometers, baro altimeters, and compass, all integrated on mobile phones such as new Android based Smartphones, it is interesting to transform the actual fusion algorithm based on modern nonlinear filtering used in those technologies and develop robust forms of these algorithms against denied GNSS environment and keep the use of such devices for important matters such as blind peoples tracking and navigation in the city, autonomous robots for multi purposes in the human life and especially to improve the quality of existing tracking systems present with millions of devices in the world market and used for Aircrafts tracking and fleet management, ships, boats, cars, and also peoples tracking for safety purposes in additional to all parallel Space applications.

Finally a novel and original theoretical and practical example on the basis of "Iridium Satellite Only Based Positioning System during GNSS missing" is giving as a real perspective for both mathematical studies and for technological development of a navigation solution on the surface of the earth in ocean, desert and even in urban environment. Analogue solution is then extrapolated to another original navigation problem devoted to astronauts and robonauts navigation on the surface of Mars planet using Phobos or Phobos/Deimos "natural satellites" instead of launching 06–08 artificial satellite on the orbit of Mars. This work has been object of 37 publications with 09 papers indexed SCOPUS and 01 paper accepted after revision in ISI THOMSON Journal.

Keywords: Kalman filter, nonlinear filtering, non-Gaussian noise, Gaussian Mixture, GPS, GNSS, Impulsive noise, MEMS.

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ACKNOWLEDGEMENTS

I would like to express my sincere gratitude and my distinguished considerations to my supervisors Professor Alexander V. Nebylov and the defunct Professor Gennadi B. Yatsevitch for guidance, time, trust and continuous support.

I would like to express my sincere gratitude and my distinguished considerations to Professor Boris T. Polyak from ICS-Moscow to be the leading Organization of my dissertation thesis.

I greatly thanks SPbSU Rector for the given opportunity to defend my PhD degree of the Program organized by the Department of Mathematics of SPbSU (Saint Petersburg State University) and the International Institute for Advanced Aerospace Technologies of Saint Petersburg State University of Aerospace Instrumentation.

This work was proposed and founded from Saint Petersburg State University of Aerospace Instrumentation (Russia).

I'm very grateful to Dr. Pau Closas from (The Centre Tecnològic de Telecomunicacions de Catalunya-CTTC) for his valuable contribution, help and collaboration.

I'm very grateful to Prof. Gonzalo-Seco Granados and Professor Robert Piché for their participation as a European members of my defense committee. I am honored.

To "my Father and my Mother", to my brothers, my wife and my son "Mohamed Idris" for their continuous support, for their trust and for being my main inspiration sources.

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PATENT

25. H. Benzerrouk, A. Nebylov, G. Yatsevitch. Localization and Navigation System for Blinds Peoples in the City. Russian Patent N°89221, issued on 08 November 2009, Moscow, Russia.

ABBREVIATIONS AND ACRONYMS

ACRLB	Approximated Cramer Rao Lower Bound
AF	Adaptive Fading
CDKF	Central Difference Kalman Filter
CEP	Circle of Error Probability
CKF	Cubature Kalman Filter
CRLB	Cramer Rao Lower Bound
DDF	Divided Difference Filter
DME	Distance Measurement Equipment
EIF	Extended Information Filter
EKF	Extended Kalman Filter
GHKF	Gauss Hermite Kalman Filter
GLONASS	Global'naya Navigatsionnaya Sputnikovaya Sistema
GNSS	Global Navigation Satellite Systems
GPS	Global Positioning System
GSACKF	Gaussian Sum Adaptive Cubature Kalman Filter
GSCKF	Gaussian Sum Cubature Kalman Filter
GSF	Gaussian Sum Filter
IEIF	Iterated EIF
IEKF	Iterated EKF
IF	Information Filter
ILS	Instrument Landing System
INS	Inertial Navigation System
KF	Kalman Filter
MEMS	MicroElectroMechanical Systems
MMSE	Minimum Mean-Square Error
MSE	Mean Square Error
pdf	Probability density function
PF	Particle Filter
RADAR	Radio Detection and Ranging
RGV	Random Gaussian Variable
RMSE	Root Mean Square Error
SBD	Short Burst Data (Iridium Telecommunication Protocol)
UAV	Unmanned Aerial Vehicle
UKF	Unscented Kalman Filter
VOR	VHF Omni Range
VR	Virtual Range beacon
TTFF	Time To First Fix

NOTATIONS

$\omega_{b}(t)$	Angular rate in the body frame
$\delta\omega_{\!\scriptscriptstyle b}(t)$	Error in angular rate
$C_b^n(t)$	Rotational Matrix from body frame to navigation frame
$\Omega(t)$	Rotational Skew Matrix
Δt	The constant rate of period
$a_b(t)$	Acceleration in the body frame
$\delta a_b(t)$	Error in the acceleration vector
g_n	Gravity field in the navigation frame
W _k	White Gaussian noise
u(k)	Input vector in discrete time
$\sigma^2_{a_b}$	Accelerometers noise variance
$\sigma^2_{\scriptscriptstyle \omega_b}$	Gyroscopes noise variance
H_k	Jacobian of observation Matrix
$\hat{x}_{k+1/k}$	Predicted state
\hat{x}_k	State estimate
Q_k	State noise Covariance Matrix
$P_{k/k-1}$	State predicted Covariance Matrix
R_k	Measurement noise Covariance Matrix
P_k	State estimate Covariance Matrix
K_k	Kalman gain
$P_{y_k x_k}$	Inter Covariance Matrix
ε	Contaminating parameter of Gaussian mixture density
σ_1^2 , σ_2^2	Variances of Gaussian mixture components
N_E	Prime vertical radius of the earth
$arphi$, λ , h	Latitude, Longitude, Altitude in the Geodetic reference system
e^2	The first eccentricity

1 INTRODUCTION

1.1 Intellectual merit

This dissertation consists on the analysis and comparison of development of robust modern non linear filtering algorithms. These robust filters are applied to original integrated navigation systems based on inertial sensors and Global Navigation by Satellite Systems GPS, GLONASS, GALILEO, BEIDOU, ... etc. In the dissertation, Part 1 provides description of the integrated navigation sensors and mathematical models involved based on original non linear information filters. Part 2 provides general description of modern non linear filters starting from Kalman filter [63, 64, 75] and its extended form [5, 33, 45, 46, 48, 75-76], then to Sigma Point Kalman Filters (SPKF) [17, 23, 54, 65–67, 72], Divided Difference Filters (DDF) [38, 44, 53, 54], Gauss Hermite Kalman Filter (GHKF) [36, 58-59] and Cubature Kalman Filters (CKF) [36, 68]. Estimation accuracy and asymptotic stability is analyzed through multiple comparisons with Mean Square Error (MSE) and Cramer Rao Lower Bound (CRLB) as reference criteria [60]. Part 3 is divided in three important sections; with three contributions. First, adaptive fading algorithm is applied to non linear filters described in this dissertation with accuracy estimation analysis of different non linear state space models; the second point consist on simulation of integrated navigation system in denied environment especially during denied GNSS using outliers model based on impulsive noise [7, 12, 16, 24-27, 31-32, 34-35, 40-41, 62]. Gaussian mixture filtering approaches are applied to the modern filters EKF, SPKF, CKF and GHKF, these are transformed into Gaussian sum filters [1, 11, 26–27, 33, 37, 47, 50, 55, 61, 73, 77]. Then, the third section of Part 3 provides the hybrid robust non linear filters based on Adaptive Fading and Gaussian mixture CKF and GHKF algorithms. In part 3, non linear filters variants are analyzed, derived and compared on the basis of MSE/RMSE. Part 4 is dedicated to integrated navigation systems and provides an overview of the actual development of advanced aerospace technologies. Description of modern sensors is well presented and integration architectures are also well investigated with several existing and expected designs, especially a novel design for

blind people localization and navigation in the city. Part 5 contains three applications of developed and investigated robust techniques proposed in this dissertation. Gaussian mixture filters are applied first to integrated navigation systems INS/GNSS and other sensors for several navigation for UAV navigation (Part 1, Part 2 and Part 3), then, application to robot navigation based on multiple sensors fusion based on decentralized non linear Information filter [5, 6, 20, 29-30, 46] (Cubature Information Filter CIF) proposed with its non Gaussian variant based on Gaussian mixture filtering in denied GNSS environment. Then, Original Pedestrian Navigation System especially for blind peoples is developed, The application dedicated to blind peoples localization and guidance in the city is based on GPS/GLONASS/Compass and robust data fusion [10, 15, 39], with special correction and self calibration of electronic compass done using road orientation; and magnetic declination of streets in the city. As a final point, original and innovative positioning and navigation system based on Iridium Satellite only based signal processing is carried out with mathematical and practical proof of impulsive noises existence during Iridium data processing. A mathematical model for robust estimation then is proposed to solve the problem of navigation in denied GNSS or during missing positioning signals [14, 18, 19, 9]. This work has been developed, tested and patented in Russian Federation. Finally, a general conclusion about the effectiveness of the proposed algorithms is summarized, carrying out very interesting prospects for future works.

1.2 Goal of the work

The goal of this work consists on the development of robust non linear filters for integrated navigation systems based on inertial sensors and external aids such as GNSS. Denied environment has been privileged with high initialization error and non Gaussian measurement noises such as in interfered environment. These non linear filtering algorithms are applied to UAV navigation, robot navigation, pedestrian and land navigation in denied GNSS environment. The robust non linear filters proposed in this dissertation are based on modification of EKF, SPKF, DDF, GHKF and CKF, most likely known as respectively Taylor based linearization filter, Unbiased filters, interpolation filters, Gauss Hermite based Quadrature filters and Cubature based Kalman filter.

1.3 The main results

The set goal was achieved by solving the following problems:

1. Comparison between the direct filtering approach for sensor fusion and the indirect filtering approach based on inertial error model.

- 2. MSE/RMSE Analysis of modern non linear filters for time series estimation problem then to solve original integrated navigation problems in aerospace.
- 3. Comparison between non linear filters in specific cases of high initialization error of the state estimate, based on adaptive fading algorithm were applied to EKF, SPKF ,DDF, GHKF and CKF.
- 4. Development of Gaussian mixture non linear filters with application to INS/GNSS integrated navigation system in denied GNSS environment affected by alpha-Stable noises defined by the symmetric Gaussian mixture pdf with time varying variances of Gaussian components.
- 5. Selection of the CKF algorithm as the best estimator with the synthesis of its Information Filter and its Gaussian mixture variant with application to multiple sensor fusion for UAV, 2D robot navigation and 2D Pedestrian navigation in non Gaussian environment.
- 6. Original solution for Blind people's localization and navigation in the city was developed and has been experimented, then patented in Russian federation. In addition, real iridium based tracking system has been experimented to tests nonlinear filters developed in this dissertation and additional geometrical algorithms.

1.4 Method of Investigation

In the course of dissertation research, the following methods are used: the methods of system analysis, estimation theory, non linear estimation and non linear filtering techniques, adaptive fading factors, Gaussian mixture approaches for non Gaussian filtering problems were investigated with application to INS/GNSS integrated navigation system. The calculation or the lower bound of estimation in the case of non linear systems with Gaussian and non Gaussian noises are also subject of mathematical development. MATLAB Software was the principal soft-tool used in this thesis in addition to MEMS sensors, compass, GPS receiver. AT commands and SBD iridium protocols have been used during the multiple experiments.

1.5 Scientific Novelty

Scientific novelty of the concluded researched consists of the following:

1. Direct filtering applied to integrated navigation system INS/GNSS is a comparative solution to the indirect filtering based on inertial error dynamical system and should be the basis of the future implementation of integrated systems especially in denied GNSS environment.

- 2. The Comparison in the accuracy and stability between different non linear filtering approaches has been computed in dissertation, with accuracy and computational complexity analysis.
- 3. Adaptive fading usually applied to linear filters has been extended to modern non linear filters SPKF, GHKF and CKF. Adaptive fading factor is introduced differently during sequential measurement update step, under high error state initialization. During simulation tests, divergence of EKF was observed as an unusual case which has determines its limitations. Fast convergences of the adaptive fading non linear filters have been achieved.
- 4. Gaussian mixture EKF as a reference algorithm, is compared to Gaussian mixture SPKF "DDF, UKF, CDKF, GHKF and CKF", in denied GNSS environment with impulsive measurement noises. Then, on the basis of the symmetric distribution, our results are compared to analog researches based on Huber estimators presented in particular and recent scientific publications in this field at the American Institute for Aeronautics and Astronautics (2007–2011). Finally, decentralized data fusion based on the proposed Gaussian Mixture Cubature Information Filter is increasing accuracy and provide the best sensors fusion for real time implementation according time complexity and accuracy.
- 5. Original solution of localization and guidance for Blind peoples in the city was suggested based on well known sensors, GPS/GLONASS and electronic compass, integrated with robust filters developed in dissertation. Russian Patent N° 89221, issued on 08 November 2009, Moscow, Russia.
- 6. The proof of the necessity to develop new performing nonlinear filters against non-Gaussian noises and especially against impulsive noise during real tests has been stated.

1.6 Practical Value

The results of dissertation may serve as basis for improving accuracy and convergence of non linear estimation algorithms in presence of non Gaussian noises, and also methods to formulate Gaussian mixture non linear filters applied to integrated navigation systems in Aerospace. The results obtained in this thesis allows for the following:

1. Optimize data fusion problems in robotics, aeronautic, space and other applications using novel formulations of non linear information filters proposed on the basis of modern approaches UKF, GHKF and CKF.

- 2. Attractive results are the fast convergence of the proposed integration techniques in real applications under high initial state uncertainties such as for robotic applications, UAV navigation, marine navigation...etc.
- 3. Ensuring robust integrated navigation system for airplane under low and high unintentional interferences such as: from VOR/DME, RADAR, ILS, TV, UHF,GSM, Iridium satellite telecommunication frequencies, Tracking applications, pedestrian localization and navigation...etc
- 4. Implementation of the proposed algorithms on smartphone Android such as GALAXY Smartphones, iPhones, iPad, iPod,...etc for people localization and navigation using the Android SDK (Software Development Kit) and the integrated sensors such as accelerometers, magnetometers, gyroscopes, cameras,...etc
- 5. Real localization system and guidance of Blind peoples in the city is also a real potential and challenge to achieve by the proposed techniques. Russian Patent N°89221, issued on 08 November 2009, Moscow, Russia.
- 6. Proposal of novel and original positioning and navigation system based on iridium satellite network with new initialization algorithms.

1.7 Implementation results

The results deduced and obtained in this dissertation have been implemented and tested at IIAAT on real integrated navigation system IMU/Magnetometers/GPS/Baroaltimeter "Open Pilot Board" and are subject of interest in testing and implementation at JSC Russian Navigation Technologies. These results are coherent, if we compare with other research works results obtained by specialists in this field at AIAA between 2007–2011 [42]. The main interesting in parallel appraoch is the ability to implement and obtain such results in real time implementation on processors and FPGA circuits, it is more suitable for parallel programmation than other techniques such as implementing Huber estimators.

1.8 Approbation of work

Main results of dissertation were presented and discussed at several conferences and reviewed in famous journals in this field. At IEEE Aerospace Conference,USA-2010, 17th ICINS at Saint Petersburg-2010, at 1st World Space conference in Germany-2010, at IFAC Aerospace congress , Nara, Japan,2010 , at IEEE Aerospace Conference USA-2011, 17th ICINS at Saint Petersburg-2011, IFAC international Congress, Milano-2011, at IFAC AGNC, India-2012, at IEEE Aerospace Conference USA-2012, at Saint Petersburg-2012, at Kazan Journal "Actual problems in Aviation and

Aerospace Systems" 2012, Journal of Signal and Information Processing, 2012, at Cybernetics and Physics Journal 2012, Inside GNSS magazine and Inside GNSS column 2012, in addition to Patent N°89221, issued on 08 November 2009, Moscow, Russia. The main results have been presented at seminar of Institute of Control Sciences in Moscow on 27 November 2012 and also at seminar of SUAI on 07 December 2012. In 2013, Seven (07) publications have been released in indexed and abstracted journals, Journal of Signal and Information Processing - JSIP Vol.4 No.4, November 2013, Journal of Sensor Technology JST - Vol.3 No.4, December 2013. International Review of Aerospace Engineering – (IREASE), December 2013 (Vol. 6 N. 6). Journal of the Moscow Aviation Institute, Vestnik, Vol. 20, No.5, December 2013. International Journal on Information Technology (IREIT) January 2014 (Vol.2 N.1). Last results have been accepted at IFAC Congress 2014, and ICINS- 2014 Elektropribor, St Petersburg. At the International IFNA-ANS scientific Journal, "Problems of nonlinear-analysis in engineering systems", № 1 (41), vol. 20, 2014, 22– 33. UGATU Vestnik (VAC) Journal,2014 and submitted into a revised form at the Journal GPS Solutions-Springer Edition 2014.

Structure and volume of dissertation. The dissertation work consists on the main content, conclusion, references and appendixes. The work is presented in 63 pages of basic text, figures, references and appendixes, in addition to 04 most important publications.

2. THE MAIN CONTENT

2.1 Introduction

The statement of estimation problems especially non linear filtering algorithms applied in aerospace are investigated and described. Original integrated navigation problems are treated and solved using different filtering methods. A state of the art of INS/GNSS integrated navigation system [21, 22, 70] and aerospace sensors fusion [10, 56] based on Filtering techniques and especially on non linear filtering is also overviewed. Original solutions are proposed and developed with well evaluated observed results. Repartition of dissertation work during last three years have been also enumerated and commented. In low cost integrated system, the Kalman filter is generally used to combine the outputs of IMU, linear accelerations and angular rate for strap down configuration with kinematical model of vehicle. Global Position System (GPS) outputs such as position and velocity are used to correct Inertial Navigation System (INS) errors growing in time. This correction is possible using Kalman filter in the linear model case and extended Kalman filter in the nonlinear case, which is the most useful filter for integrated navigation system problems.



FIGURE 1 Direct Filtering Development for Integrated Navigation System

This is why it is called an indirect mode [23, 56, 70]. For the second approach, i.e. the direct mode, the state vector is estimated directly via nonlinear estimator like EKF or other nonlinear filter like Sigma-Points Kalman filters.



FIGURE 2 Direct Filtering — Indirect Filtering for Integrated Navigation System

The main problem in inertial navigation system is the bias and drift of the accelerometers and gyroscopes. In fact the problem of filtering is followed by control problem resolution in order to improve and realize more accurate control algorithm based on more accurate estimates. To solve this problem, different approach can be used such as indirect and direct mode and different filters can be applied [51,56,70]. The first approach means estimation of the state vector errors using linear Kalman filter and summing these values at the output of the inertial system design, see [Benzerrouk et al., 3].

Direct combination approach

Within an integrated navigation system, the filter can be configured either as a direct or indirect form depending on the types of sensors and the complexities of the system. In a direct configuration, the filter directly estimates the states of interest. It typically constitutes a main functional block within the system performing both the deadreckoning and the observation fusion. In the indirect formulation, the filter estimates the error quantities of the desired states, and applies this error to the external deadreckoning loop for correction, hence it estimate the state indirectly.



FIGURE 3 Inertial and Navigation (N, E, D) frames

The thesis dissertation presents a new method for a low-cost strapdown-IMU/GPS combination [39, 52], with data fusion for the determination of 2-D/3-D components of position (trajectory), velocity and attitude angles. In this approach called "Direct filtering", it is assumed that earth rotation and gravity variations are neglected, with the use of low gyroscope sensitivities of the low-cost IMU and due to the relatively small volume and/or the surface of the mobile trajectory. It is possible to define:

Inertial frame (i), Earth Fixed frame (e), Navigation frame (n), Body frame "body" (b).

The scope of this thesis was to test the feasibility of an integrated navigation system based on multiple low cost sensors fusion and to develop adaptive and robust nonlinear filters for such a combination in denied GNSS environments. The local reference system NED (North, East, Down) (denoted here with n) is assumed in this thesis to be an inertial frame (see Fig. 3), for more details, see [39, 70].

Inertial Kinematic model

This research focuses on the unexplored direct filtering applied to INS/GNSS integrated navigation systems with non-linear dynamic inertial kinematic models. Inertial measurement units (IMUs) typically contain three orthogonal rate-gyroscopes and three orthogonal accelerometers, measuring angular velocity and linear acceleration respectively. Ideally, the output of the rate-gyroscopes is written as:



FIGURE 4 Inertial Measurement Unit Mechanization

Angular velocity
$$\omega_b(t) = \begin{bmatrix} \omega_{bx}(t) & \omega_{by}(t) & \omega_{bz}(t) \end{bmatrix}^T$$
 (1)

In practice, however, the outputs contain errors and are written as

$$\widetilde{\omega}_{b}(t) = \omega_{b}(t) - \delta\omega_{b}(t), \quad \delta\omega_{b}(t) = \begin{bmatrix} \delta\omega_{bx}(t) & \delta\omega_{by}(t) & \delta\omega_{bz}(t) \end{bmatrix}^{T}$$
(2)

Integrating this yields the updated attitude information for the system [39, 56, 65-66],

$$\frac{d}{dt}C_{b}^{n}(t) = C_{b}^{n}(t)\Omega(t), \qquad \Omega(t) = \begin{bmatrix} 0 & -\widetilde{\omega}_{bz}(t) & \widetilde{\omega}_{by}(t) \\ \widetilde{\omega}_{bz}(t) & 0 & -\widetilde{\omega}_{bx}(t) \\ -\widetilde{\omega}_{by}(t) & \widetilde{\omega}_{bx}(t) & 0 \end{bmatrix}$$
(3)

$$C_{b}^{n}(t+\Delta t) = C_{b}^{n}(t)e^{\int_{t}^{t\Delta t}(\Delta t)dt} \approx C_{b}^{n}(t)\left(I - \frac{\sin\left(\left|\widetilde{\omega}_{b}\Delta t\right|\right)}{\left|\widetilde{\omega}_{b}\Delta t\right|}\Omega(t)\Delta t + \frac{1-\cos\left(\left|\widetilde{\omega}_{b}\Delta t\right|\right)}{\left|\widetilde{\omega}_{b}\Delta t\right|}\left(\Omega(t)\Delta t\right)^{2}\right)$$
(4)

Similarly, accelerometers outputs can be written as:

$$a_b(t) = \begin{bmatrix} a_{bx}(t) & a_{by}(t) & a_{bz}(t) \end{bmatrix}^T, \quad \widetilde{a}_b(t) = a_b(t) - \delta a_b(t), \quad (5)$$

Two integrations subsequently yield velocity and position updates as follows

Velocity integration :
$$V_{n,k} = V_{n,k-1} + \Delta t \left(\widetilde{a}_{n,k} - g_n \right),$$
 (6)

Position integration :

$$Pos_{n,k} = Pos_{n,k-1} + \Delta t \left(V_{ng,k} \right)$$
⁽⁷⁾

where g_n is the estimated gravity vector and Δt is the constan rate of period. Collectively, equations eq.3-eq.7 describe the system model. After differential and mathematical development, it is possible to write the following state space model [39] which is the basis of the future implementations:

$$x(k) = f(x(k-1), u(k), w_k)$$
 (8)

$$\mathbf{E}[w_{\mathbf{k}}] = \mathbf{0} \tag{9}$$

$$\mathbf{E}[w_{\mathbf{k}}w_{\mathbf{k}}^{T}] = Q(k) = \begin{bmatrix} \sigma_{a_{b}}^{2} & 0\\ 0 & \sigma_{\omega_{b}}^{2} \end{bmatrix}$$
(10)

$$\begin{bmatrix} \mathbf{p}_{n}(k) \\ \mathbf{v}_{n}(k) \\ \psi_{n}(k) \end{bmatrix} = \begin{bmatrix} \mathbf{p}_{n}(k-1) + \mathbf{v}_{n}(k-1)\Delta t \\ \mathbf{v}_{n}(k-1) + \left\{ \mathbf{C}_{n}^{b}(k-1) \left[a_{b}(k) + \delta a_{b}(k) + g^{n} \right] \right\} \Delta t \\ \psi_{n}(k-1) + \mathbf{E}_{b}^{n}(k-1) \left[\omega_{b}(k) + \delta \omega_{b}(k) \right] \Delta t \end{bmatrix} + \begin{bmatrix} w_{p^{n}}(k) \\ w_{v^{n}}(k) \\ w_{\psi^{n}}(k) \end{bmatrix}$$
(11)

The Jacobian matrices are used in the implementation of the extended Kalman Filter EKF or the Linearized Kalman Filter LKF, or as the basis of the derivation of the second order Kalman filter 2nd Order KF [2, 3, 15, 27, 39, 46, 48, 51, 56, 75].

GNSS position information and novel Satellite based Positioning

GNSS signal processing is much explored based on different algorithms tested more and more in real time conditions and in simulations during different denied conditions. *GPS*-GLONASS satellites also broadcast signals in the L1 and L2 sub-bands of the radio frequency spectrum as described in the literature[55, 57, 61]. It is observed in some situation several interferences from different sources for GPS and GLONASS during static and dynamic positioning. GNSS outages or outliers cause accuracy degradation, and sometimes undelivered GNSS receiver positioning. Correspondingly, we can write the observation matrix as

$$H_{GNSS,k} = h_k \left(P_{GNSS,k}, V_{GNSS,k}, A_{GNSS,k} \right)$$
(12)

$$P_{j}^{i} = \rho_{j}^{i} + c\,\delta t = \sqrt{\left(x_{j} - x^{i}\right)^{2} + \left(y_{j} - y^{i}\right)^{2} + \left(z_{j} - z^{i}\right)^{2}} + c\,\delta t \tag{13}$$

For the purpose of this thesis, the measurement model is assumed to consists of GNSS-observed position (P_{GNSS}), velocity (V_{GNSS}) for loosely coupled approach, delta range and delta rate for tightly coupled approach and attitude (A_{GNSS}); the latter derived using a multiple antenna system. Later in this thesis, a specific problem of GNSS outliers caused by impulsive noises is then considered.

It is also possible to consider the direct observation or measurement from GNSS receivers given position in Latitude, Longitude, Altitude such as described below in eq.14.

$$P_{e} = \begin{pmatrix} x_{e} \\ y_{e} \\ z_{e} \end{pmatrix}_{GPS} = \begin{pmatrix} (N_{E} + h)\cos\varphi\cos\lambda \\ (N_{E} + h)\cos\varphi\sin\lambda \\ [N_{E}(1 - e^{2}) + h]\sin\varphi \end{pmatrix}$$
(14)

In this work, the standard, non differential, civilian signal is used. It provides a lower accuracy but acceptable in low cost integrated navigation systems, is the lowest cost GPS solution is advantageous because of its availability. The standard measurement of the GPS system is the pseudo-range and the pesuedo-rate. This defines the approximate range from the user GPS receiver reference point to GPS or GNSS satellites. The pseudo-range is the true distance corrupted with differential errors specified by the following formula [23, 39]:

$$\rho_{j} = r_{j} + \delta \rho_{r,clk} + \delta \rho_{ion} - \delta \rho_{s,clk} + v_{k}$$
(15)

Where:

 r_j Pseudo-range from the user to the jth satellite Geometric range from the user to the jth satellite

 $\delta \rho_{r,clk}$ Range equivalent receiver clock bias offset from GPS system time

 $\delta\!\rho_{\scriptscriptstyle s,clk}$ Range equivalent satellite clock bias offset from GPS system time

 $\delta \rho_{im}$ Ionospheric signal attenuation error

 v_k Zero mean white noise

Note: Multiple experiences based on GPS/GLONASS receivers have been computed during this research work in order to compare the TTF and the number of satellite in visibility. The degree of nonlinearity is known as significant under a specific conditions [61, 62].

2.2 Linear and nonlinear filtering

This part provides theory and overview of linear filtering, Kalman filtering and information filtering [5, 29, 30]. Both non linear forms of information and Kalman Filter are presented and discussed. More accurate forms of EIF and EKF based on iteration in measurement step are also discussed and well described in simulation. Below, algorithm of EKF such as given in Literature which is the basis and the reference filter for all coming results:

Extended Kalman Filter Algorithm [46, 48]

Based on state space model described by :
$$\begin{cases} x_{k+1} = f_k(x_k) + w_k \\ z_k = h_k(x_k) + v_k \end{cases}$$
 (16)

and on the linearization using Taylor approximation at the first order we get the state space model given in eq.16. $F_k(.)$ is the Jacobian matrix of $f_k(.)$ and $H_k(.)$ is the Jacobian matrix of $h_k(.)$.

Initialization : \hat{x}_0 et P_0 .

Prediction :

$$\hat{x}_{k+1/k} = f_k(\hat{x}_k) \tag{17}$$

$$P_{k/k-1} = F_k(\hat{x}_k) P_{k-1} F_k(\hat{x}_k)^T + Q_k$$
(18)

Update :

$$K_{k} = P_{k/k-1} H_{k}^{T}(\hat{x}_{k/k-1}) \left[H_{k}(\hat{x}_{k/k-1}) P_{k/k-1} H_{k}^{T}(\hat{x}_{k/k-1}) + R_{k} \right]^{-1}$$
(19)

$$\hat{x}_{k} = \hat{x}_{k/k-1} + K_{k} \left[z_{k} - h_{k} \left(\hat{x}_{k/k-1} \right) \right]$$
(20)

$$P_{k} = P_{k/k-1} - K_{k} H_{k}(\hat{x}_{k/k-1}) P_{k/k-1}$$
(21)

In practice, the application of EKF, EIF depends on the nature of the process and measurements. Based on high order Kalman filter, new non linear high order information filter has been developed with its iterative variant. Novel formulations of non linear filters are derived in this first chapter. Simulations results proof superiority of the new developed algorithms "Iterative Information Filters". Below, a description of the information filtering approach which has a serious advantage of non computational complexity in the "Kalman gain calculation".

Information Filter and Non Linear Information Filters

The information filter is mathematically equivalent to the Kalman filter except that it is expressed in terms of measures of information about the states of interest rather than the direct state and its covariance estimates. Indeed, the information filter is known to have a dual relationship with the Kalman filter [63,64]. If the system is linear with an assumption of Gaussian probability density distributions, the information matrix Y(k/k), and the information state estimate $\mathbf{y}(k|k)$, are defined in terms of the inverse covariance matrix and state estimate .

$$Y(k / k) = P^{-1}(k / k);$$

$$y(k / k) = Y(k / k)x(k / k);$$
(22)

When an observation occurs, the information state contribution i(k) and its associated information matrix I(k) are given by the following expressions:

$$i(k) = H^{T}(k)R^{-1}(k)z(k);$$

$$I(k) = H^{T}(k)R^{-1}(k)H(k);$$
(23)

By using these variables, the information prediction and update equation can be derived from Kalman filter.In this thesis, the synthesis of more accurate nonlinear information filters is achieved and each of the 2nd order Kalman filter, Unscented Kalman Filter, Central Difference Kalman Filter and finally Cubature Kalman Filter have been transformed into information filters on the basis of the Extended Information Methodology [46].

SPKF-CDKF Central Difference Kalman Filter (UKF variant)

For CDKF, it is approximately the same idea as in UKF algorithm [Benzerrouk et al., 4], at the difference of the steps of propagation of the sigma points through the non linear functions of dynamic of the system and measurement equation, behind this, the non linear approximation of these functions is done using the divided differences [55, 66]. Generally, it is used the optimal value of h = 1 [65]. Initialization

$$\hat{x}_{0} = E[x_{0}] P_{x_{0}} = E[(x_{0} - \hat{x}_{0})(x_{0} - \hat{x}_{0})^{T}]$$
(24)

Pour k=1..... $^{\infty}$, t = k-1

Augmented state and Sigma points $\hat{x}_{t}^{av} = \begin{bmatrix} \hat{x}_{t} & \overline{v} \end{bmatrix}$ (25)

$$P_t^{av} = \begin{bmatrix} P_{x_{k-1}} & 0\\ 0 & R_v \end{bmatrix}$$
(26)

$$\chi_t^{av} = \begin{bmatrix} \hat{x}_t^{av} & \hat{x}_t^{av} + h\sqrt{P_t^{av}} & \hat{x}_t^{av} - h\sqrt{P_t^{av}} \end{bmatrix}$$
(27)

Sigma points propagation through the process

$$\chi_{k/t}^{x} = f\left(\chi_{t}^{x}, \chi_{t}^{v}, u_{t}\right)$$
(28)

$$\hat{x}_{k}^{-} = \sum_{i=0}^{2L} \omega_{i}^{m} \chi_{i,k/t}^{x}$$
(29)

$$P_{xk}^{-} = \sum_{i=1}^{L} \left[\omega_{i}^{c1} \left(\chi_{i,k/t}^{x} - \chi_{i+L,k/t}^{x} \right)^{2} + \omega_{i}^{c2} \left(\chi_{i,k/t}^{x} + \chi_{i+L,k/t}^{x} - 2\chi_{0,k/t}^{x} \right)^{2} \right]$$
(30)

Measurement sigma points

$$\hat{x}_{k/t}^{an} = \begin{bmatrix} \hat{x}_k^- \ \overline{n} \end{bmatrix}$$
(31)

$$P_{k/t}^{an} = \begin{bmatrix} P_{x_k}^- & 0\\ 0 & R_n \end{bmatrix}$$
(32)

$$\chi_{k/t}^{an} = [\hat{x}_{k/t}^{an} \quad \hat{x}_{k/t}^{an} + h\sqrt{P_{k/t}^{an}} \quad \hat{x}_{k/t}^{an} - h\sqrt{P_{k/t}^{an}}]$$
(33)

1. Update

$$Y_{k/t} = h\left(\chi_{k/t}^{x}, \chi_{k/t}^{n}\right)$$
(34)

$$\hat{y}_{k}^{-} = \sum_{i=0}^{2L} \omega_{i}^{m} Y_{i,k/t}$$
(35)

$$P_{\tilde{y}_{k}} = \sum_{i=1}^{L} \left[\omega_{i}^{c1} \left(Y_{i,k/t} - Y_{i+L,k/t}^{-} \right)^{2} + \omega_{i}^{c2} \left(Y_{i,k/t} + Y_{i+L,k/t}^{-} - 2Y_{0,k/t}^{-} \right)^{2} \right]$$
(36)

$$P_{y_k x_k} = \sqrt{\left(\omega_1^{c_1} P_x^{-}\right)} \left[Y_{1:L,k/t} - Y_{L+1:2L,k/t} \right]^T$$
(37)

$$K_{k} = P_{x_{k}x^{k}} P_{\bar{y}k}^{-1}$$
(38)

$$\hat{x}_{k} = \hat{x}_{k}^{-} + K_{k} \left(y_{k} - \hat{y}_{k}^{-} \right)$$
 (39)

$$P_{x_k} = P_{x_k}^- - K_k P_{\widetilde{y}_k} K_k^T \tag{40}$$

Avec h=
$$\sqrt{3}$$
, $\omega_0^m = (h^2 - L)/h^2$, $\omega_i^m = \frac{1}{2h^2}$, $\omega_i^{c1} = \frac{1}{4h^2}$, $\omega_i^{c2} = (h^2 - 1)/4h^4$ (41)

Sigma points Kalman filters (SPKF) introduced by [65] both Unscented filters (UKF) and central difference Kalman filters (CDKF) mean the SPKF. In this case, it is not the non linear function which is estimated, but the RGV, and the density of probability using a deterministic sigma points to estimate at the first and the second order the moment of the RGV, so, the means and the covariance of the state vector can be estimated better than by the EKF, because the accuracy of these kind of estimators is the second and the third order of Taylor development.

As presented in the previous section, the algorithm use the second order polynomial interpolation of Stirling's polynom and are defined two other matrix comparing with the 1st order divided difference filter DD1 and the 2nd Order DDF [53].

The 2nd order Divided difference filter DD2 [53]

The DDF2 algorithm can also be described in the unified way used and demonstrated by replacing the first-order prediction formulas for the state and covariance with the second-order ones. The proposed algorithm, referred to as the divided difference filter (DDF) proposed by [53] is an efficient extension of the Kalman Filter for nonlinear systems. The DDF is described as a sigma point filter (SPF) in a way where the filter linearizes the nonlinear dynamic and measurement functions by using an interpolation formula through systematically chosen sigma points. The linearization is based on polynomial approximations of the nonlinear transformations that are obtained by Stirling's interpolation formula, rather than the derivative-based Taylor series approximation [38, 42, 71].

Particle Filter PF

The particle filter is a sequential Monte Carlo algorithm, i.e. a sampling method for approximating a distribution that makes use of its temporal structure. A "particle representation" of distributions is used, for more details, see [24, 32, 41, 46, 58, 73].

2.3 Modern nonlinear filtering

Modern nonlinear filtering algorithms are described by historical chronology of non linear filtering theory development. Multiple variants were developed based on high order Taylor approximation and iterative form such as well described methods in chapter1. In 1997, new variants of non linear based Kalman filter algorithms were developed and are described in detail in this chapter. Parallel solutions more accurate than EKF but still sub optimal non linear filters called Sigma Point Kalman Filters including two variants: (Unscented Kalman Filter) and (Central Difference Kalman Filter). Using different philosophy such as in particle filtering, these algorithms compute deterministic sample points instead of random samples in order to estimate the mean and the covariance of the random Gaussian variable RGV. Chapter 2 describes in detail algorithms of SPKF (CDKF) used in this dissertation as a second more accurate references than EKF. Finally most recent algorithm (2009) called Cubature Kalman Filter CKF is compared with other SPKF and EKF with extended use in denied GNSS environment with non Gaussian noise, thus, Gaussian Mixture CKF-GMCKF is developed and compared to SPKF and CKF.

Cubature Kalman Filter Algorithm (Haykin 2009)

1. Draw cubature points ξ_i , $I = 1, 2, ..., 2n_x$ from the intersections of the n-dimensional unit sphere and the Cartesian axes. Scaled by $\sqrt{n_x}$. The cubature based Gaussian filter algorithms use cubature rules of the form:

$$I(f) \approx \sum_{i=1}^{m} \omega_i f(\xi_i)$$

to approximate the integral of the form:

$$\int g(x) \mathsf{N}_{\Sigma}^{\mu}(x) dx = \frac{1}{\sqrt{\pi^n}} \int g(\sqrt{2\Sigma}x + \mu) e^{-x^T x} dx \tag{42}$$

We can write then:

$$\xi_{i} = \begin{cases} \sqrt{n}_{x} e_{i} \\ -\sqrt{n}_{x} e_{i-n} \end{cases} \text{ for } i=1,..., n_{x}, i=n+1,..., 2n_{y}$$

See [36, 37, 59, 68] for more details. Use the Cubature points for propadation in the algorithm presented in Gauss Hermite Kalman Filter which is defined by eq. 51–62, see [Benzerrouk et al., 1].

Gauss Hermite Kalman Filter

The basic idea is that a KF can optimally deal with these states such as in the classic linear integration of INS and GNSS, while reducing the dimension of the state-space that the GHQKF has to explore [36, 59]. We use the direct integration approach modelling, with separation of position "linear" and Velocity, Attitude "non linear". Thus, one can describe Gauss-Hermite Quadrature Kalman Filter by the following:

We consider the weighted integral of function f(x) over the interval (i, j):

$$I(f) = \int_{i}^{j} W(x) f(x) dx$$
(43)

With W(x) is a weight function almost positive or equal o zero in a few points. An npoints numerical quadrature "integration" is I(f) approximated by the following formula:

$$I(f) \approx \sum_{k=1}^{n} \omega_l f(\xi_l)$$
(44)

In this approximation, ξ_l are the quadrature points and ω_l represent the corresponding weights. given *n* distinct quadrature points, one can calculate the weights ω_l using the moments M_i integral equation given below :

$$M_{i} = \int_{i}^{j} x^{i} W(x) dx, \quad \text{for} \quad i \in \{0, 1, \dots, (m-1)\}$$
(45)

And by solving the Vandermonde system of equations described below:

$$\begin{pmatrix} 1 & 1 & \dots & 1 \\ \xi_1 & \xi_2 & \dots & \xi_m \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \ddots & \cdot \\ \xi_1^{m-1} & \xi_2^{m-1} & \dots & \xi_m^{m-1} \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \cdot \\ \cdot \\ \vdots \\ \omega_m \end{pmatrix} = \begin{pmatrix} M_0 \\ M_1 \\ \cdot \\ \vdots \\ M_{m-1} \end{pmatrix}$$
(46)

The non linear system described before is known as the system of moment equations. According Gauss-Hermite rule, the weight solutions are chosen as a standard Gaussian density with zero mean and unit covariance components. A complex and difficult solution to determine the weight function is to solve the non linear system equations. Another way has been chosen as the zeros of the m-th order Hermite polynomial [36, 59]. Since the Hermite polynomials are distinct, it is known that the determinant of the eq.46 is non zero due to Vandermonde determinant property.

The vector $(\omega_1, \omega_2, ..., \omega_m)$ is unique solution. Thus, for an *m*-point quadrature rule, the quadrature is exact for all polynomials that less or equal than (2m-1). Then, when considered a random variable x having a Gaussian probability density $\eta(x;0,1)$. The expected value of the function f(x) is approximated by the following expression:

$$E(f(x)) = \int_{R} f(x)\eta(x;0,1)dx$$
(47)

One can construct the quadrature points such as given in the following paragraph. This step is the key of the Gauss-Hermite Based Kalman Filter compared with Cubature rule based Kalman filtering and previous Sigma Point Kalman Filtering approaches. The difference in the determination of the quadrature implies the difference in the covariance and state estimation which will be developed later in the next sections.

Gauss-Hermite Quadrature Points generation

We consider the matrix J as a symmetric tridiagonal with zero diagonal elements. Let us write the following description:

$$J_{i,i+1} = \sqrt{i/2}, \quad 1 \le i \le (m-1)$$
(48)

Then the quadrature points ξ_l is determined by the following formula: $\xi_l = \sqrt{2}\varepsilon_l$, where ε_l is the l-th eigenvalue of the tridiagonal matrix *J*. The corresponding weight $\omega_l = (e_l)_1^2$ where e_l is the first element of the *i*th normalized eigenvector of *J*. For a Hermite polynomial of order *m*, *mⁿ* Gauss Hermite quadrature points are generated. As in the most case of non linear estimation problems such as attitude estimation for UAV or position, velocity and attitude estimation, the integral in eq.49 achieves Multidimensional quadrature rule formula by the recursive application of its computation and expectation as given below:

$$E(f(x)) = \int_{R_{nx}} f(x)\eta(x;0,I_{nx})dx$$
(49)

$$\approx \sum_{l_{nx}=1}^{m} \omega_{l_{nx}} \dots \sum_{l_{1}=1}^{m} \omega_{l_{1}} f\left(\xi_{l_{1}} \dots \xi_{l_{n_{x}}}\right)$$
$$\approx \sum_{l=1}^{m^{n_{x}}} \omega_{l} f\left(\xi_{l}\right).$$
With $\xi_{l} = \begin{bmatrix} \xi_{l_{1}} & \dots & \dots & \xi_{l_{n_{x}}} \end{bmatrix}^{T}$ and $\omega_{l} = \prod_{j=1}^{n_{x}} \omega_{l_{j}}$. (50)

For more understanding and details about the improvement of the quadrature rules and the Gauss-Hermite Qudarature and the relation with Bayesian filtering, one can read [73].

After the description of the quadrature points generation using algorithm 1 given in [Benzerrouk et al., 1], one can summarize the GHQKF algorithm such as given in the next paragraph.

The following subsection describes the GHQKF algorithm following analog steps as defined in the Cubature Kalman Filter CKF, with prediction step and measurement update. For **Cubature Kalman Filter** $m = 2n_x$.

Gauss Hermite Qudarature Kalman Filter $m = p^{n_x}$ with p the degree of Hermite polynom

Time Update step

1. Assume at time k that the posterior density function is known. Factorize

$$P_{k-1/k-1} = \sqrt{P_{k-1/k-1}} \left(\sqrt{P_{k-1/k-1}} \right)^T$$

2. Evaluate the Quadrature points $\{X_{l,k-1/k-1}\}_{l=1}^m$ as given below

Propagate Quadrature points. The matrix square root is the lower triangular Cholesky factor.

$$X_{l,k-1/k-1} = \sqrt{P_{k-1/k-1}} \xi_l + \hat{x}_{k-1/k-1}$$
(51)

3. Evaluate the Quadrature points with dynamic model function:

$$\chi_{l,k/k-1}^{*} = f(X_{l,k-1/k-1}, u_{k-1}, k-1)$$
⁽⁵²⁾

4. Estimate the predicted state mean:

$$\hat{x}_{k/k-1} = \sum_{l=1}^{m} \omega_{l} \chi_{l,k/k-1}^{*}$$
(53)

5. Estimate the predicted error covariance:

$$P_{x_{k}}^{-} = \sum_{l=1}^{m} \omega_{l} \chi_{l,k/k-1}^{*} \chi_{l,k/k-1}^{T} - \hat{x}_{k/k-1} \hat{x}_{k/k-1}^{T} + Q_{k-1}$$
(54)

Measurement Update step

Factorize
$$P_{k/k-1} = \sqrt{P_{k/k-1}} \left(\sqrt{P_{k/k-1}} \right)^T$$

- 1. Generate Quadrature points $\{X_{l,k/k-1}\}_{l=1}^m$ (as in step 1).
- 2. Propagate the Quadrature points.

$$X_{l,k/k-1} = \sqrt{P_{k/k-1}} \xi_l + \hat{x}_{k/k-1}$$
(55)

3. Evaluate the quadrature points with the measurement model.

$$Y_{l,k/k-1} = h(X_{l,k/k-1})$$
(56)

4. Estimate the predicted measurement:

$$\hat{y}_{k/k-1}^{-} = \sum_{l=1}^{m} \omega_l Y_{l,k/k-1}$$
(57)

5. Estimate the innovation covariance matrix.

$$S_{k/k-1} = \sum_{l=1}^{m} \omega_l Y_{l,k/k-1} Y_{l,k/k-1}^T - \hat{y}_{k/k-1} \hat{y}_{k/k-1}^T + R_k$$
(58)

6. Estimate the cross covariance matrix.

$$P_{xy,k/k-1} = \sum_{l=1}^{m} \omega_l X_{l,k/k-1} Y_{l,k/k-1}^T - \hat{x}_{k/k-1} \hat{y}_{k/k-1}^T$$
(59)

7. Estimate the Kalman gain.

$$W_{k} = P_{xy,k/k-1} S_{k/k-1}^{-1}$$
(60)

8. Estimate the update state.

$$\hat{x}_{k} = \hat{x}_{k/k-1} + W_{k} \left(y_{k} - \hat{y}_{k/k-1} \right)$$
(61)

9. Estimate the error covariance

$$P_{k/k} = P_{k/k-1} - W_k S_{k/k-1} W_k^T$$
(62)

In this work, the use of novel Quadrature Kalman Filter (QKF) called Gauss Hermite Kalman Filter (GHKF) is applied to INS/GPS/GLONASS integration problem in Gaussian and non Gaussian noise environment [1, 45, 55, 57, 69, 78, 79]. Especially Marginalized QKF (M-QKF) is proposed and developed. Navigation state including linear part with position integration and non linear part with velocity and attitude angles integration is proposed as an appropriate model to GHKF and CKF algorithms [65–66]. In this dissertation, the limits of these algorithms have been determined when the model of measurement noises changes to non Gaussian. Especially in denied GNSS environment, noises are supposed to follow the alpha stable symmetric distribution. To solve this problem, Gaussian mixture filtering method has been chosen instead of Huber based estimation method [34, 35, 42]. These algorithms are applied to integrated navigation systems low cost inertial sensors.



FIGURE 5 Guidance Navigation and Control System for UAV

Multiple trajectories have been simulated in a closed form in order to appreciate how can each filtering algorithm tracks the reference trajectory especially compared with IMU deviation after a few minutes.

Results of GHKF-CKF- CDKF with 5^{nx} Quadrature points (5⁹ points)

During these new simulations, the number of Gauss-Hermite Sigma Points has been increased to 5^{nx} Quadrature points compared with Sclaed GHKF of 3^{nx} Quadrature points.



FIGURE 6 MSE yaw angle 5^{nx} Quadrature points — MSE (Scaled 3^{nx} Quadrature points)

In Fig.6, it is possible to observe the classification carried out from simulation results with two main classes related to MSE estimation; EKF and UKF provides the approximated same results in attitude estimation based on Euler angles, however CDKF, CKF and GHKF are the best estimators with additional smother classification, GHKF presents smother characteristics which can be explained by the use of higher number 5^{nx} Quadrature points of Sigma Points. It is the only filter with high error estimation stability and with the best accuracy achievement, see [2, 3, 4, 8].

Results of GHKF-CKF-CDKF with 3^{nx} Quadrature points based on Scaled GHKF (3⁹ points)

A scale factor is introduced in the prediction and estimation step such as in adaptive fading factor and has improved the accuracy of GHKF estimation which carries out three classes of non linear filters well distinguished according simulation results shown in Fig.6. During all simulations presented, it is clear that in the accuracy order, we obtain the following nonlinear filters: EKF, UKF, CDKF, CKF, GHKF. Cramer Rao lower born [1, 46, 58, 60] has been simulated as the minimum estimation variance in order to compare all non linear filtering approaches proposed in this work.

2.4 Innovation based adaptive fading

This section contains adaptive fading based non linear filters presented in Part 2 [48– 49, 80]. Adaptive techniques are developed in dissertation and are demonstrated in simulations. First, adaptive fading technique was selected and extended from the linear Kalman filter to the other four non linear variants; EKF, SPKF, DDF, and CKF. Sub optimal and optimal adaptive innovation based fading factors are calculated and applied to ensure higher tracking consiste. Only sequential measurement adaptive fading update is applied in order to produce efficient non linear filters with reduced time complexity. The calculation of adaptive fading factor is given below: $\lambda(k)$ is called the suboptimal fading factor [Benzerrouk et al., 2]. Suboptimal fading factor is determined and provided in dissertation, we propose the fading factor given by:

$$\lambda_{i,k} = \frac{tr[\eta V_k - \varepsilon R_k]}{tr[P_{yy}]} = \begin{cases} \lambda_{i,k}, & \lambda_{i,k} > 1\\ 1, & \lambda_{i,k} \le 1 \end{cases}, \quad V_k = \begin{cases} \frac{v_0 v_0^T}{\rho V_{k-1} + v_k v_k^T} \\ 1 + \rho \end{cases}, k \ge 2$$
(63)

$$P_{x_{k}}^{-} = \frac{\lambda_{i,k}}{2n} \sum_{i=1}^{2n} \chi_{i,k/k-1}^{*} \chi_{i,k/k-1}^{T} - m_{k/k-1} m_{k/k-1}^{T} + Q_{k-1}$$
(64)

$$S_{k/k-1} = \frac{\lambda_{i,k}}{2n} \sum_{i=1}^{2n} Y_{i,k/k-1} Y_{i,k/k-1}^T - \hat{y}_{k/k-1} \hat{y}_{k/k-1}^T + R_k$$
(65)

$$P_{x_{k},y_{k}/k-1} = \frac{\lambda_{i,k}}{2n} \sum_{i=1}^{2n} X_{i,k/k-1} Y_{i,k/k-1}^{T} - m_{k/k-1} \hat{y}_{k/k-1}^{T}$$
(66)

with $0 < \rho < 1$ as the pre-selected forgetting factor, it has to be tuned according the process covariance noise apriori. When this factor is calculated, it is used into the measurement update step which occurs by sequences in all non linear algorithms described previously. In the dissertation; algorithms of the Adaptive Fading EKF (AFEKF), Adaptive Fading SPKF, Adaptive Fading DDF and Adaptive Fading CKF are applied to integrated navigation systems.

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FIGURE 7 MSE Down velocity estimation (m/s)-zoom

In the dissertation, all these new formulations based on adaptive fading algorithms are simulated and applied to non linear time series estimation in order to test and proof those efficiencies, accuracy and time of convergence. Results in Fig.7 show unusual cases of divergence of extended Kalman Filter EKF, with superiority of the proposed adaptive forms.

Adaptive fading based nonlinear filtering

The time-varying suboptimal scaling factor is incorporated [48,77,80], for on-line tuning the covariance of the predicted state, which adjusts the filter gain, and accordingly the adaptive filter is developed. The suboptimal scaling factor in the time-varying filter gain matrix is given by:

$$\lambda_{k} = \max\left\{1, \frac{tr[N_{k}]}{tr[M_{k}]}\right\}$$
(67)

Some other choices of the factors are also used, where tr[] is the trace of matrix. The parameters are given by

$$N_k = \gamma V_k - \beta R_k - H_k Q_k H_k^T$$
(68)

$$\boldsymbol{M}_{k} = \boldsymbol{H}_{k}\boldsymbol{\phi}_{k}\boldsymbol{P}_{k}\boldsymbol{\phi}_{k}^{T}\boldsymbol{H}_{k}^{T}$$

$$\tag{69}$$

$$V_{k} = \begin{cases} v_{0}v_{0}^{T}, k = 0 \\ \left[\rho V_{k-1} + v_{k}v_{k}^{T}/1 + \rho\right], \dots, k \ge 1 \end{cases}$$
(70)

The key parameter in the adaptive filtering described in this section is the fading factor matrix λk , which is dependent on three parameters, including (1) α_i ; (2) the forgetting factor (ρ); (3) and the softening factor (β). These parameters are usually selected empirically. Multiple simulations have been computed and interesting observations have been carried out. In simulated cases, EKF, UKF, CDKF, DDF and CKF converge but require high duration of time between 10–20 sec. However, AF-EKF, AF-SPKF, AF-DDF and AF-CKF converge in less than one second. Below simulation results of adaptive fading non linear filters against large initialisation error to integrated navigation system INS/GNSS, see Fig.8.



FIGURE 8 3D trajectory estimation — North distance estimation

In this part, we have improved SPKF using adaptive fading factors with fast convergence and high accuracy as new results. Another problem in filtering theory and practice was to overcome the Gaussian assumption of noises models of the measurements and in some cases in both equations "system and measurement".

2.5 Robust Cubature Kalman Filters-"Gaussian Mixture-CKF"

In many estimation problems, it is appropriate to consider non-Gaussian noise distributions to model possible outliers or impulsive behaviors in the measurements, especially during navigation missions. In this paper, we considered a nonlinear filtering problem with a Gaussian process noise and a Gaussian mixture distributed measurement noise. Both processes' statistics parameters are assumed to be known. Within this assumptions, we present a filtering method based on a SPKF, CKF and GHKF that use Gaussian Sum Filter GSF property and accounts for such heavier distribution tail and its varying parameters.

Gaussian Mixture and Impulsive Noise

In dissertation all robust filters are simulated and compared on the basis of the Mean Square Error of estimation and compared to approximated Cramer Rao Lower Bound. By the way, some important results are shown in Fig.9, providing much better and stable robust estimation. Model of measurement noise is given by the mathematical equation of the density of probability p(x). In this dissertation, the challenge was to provide estimates of the state of dynamical systems when nonlinear system coupled with non-Gaussian measures are assumed to follow the following pdf:

$$p(x) = \frac{1-\varepsilon}{\sqrt{2\pi\sigma_1^2}} \exp\left(-\frac{x^2}{2\sigma_1^2}\right) + \frac{\varepsilon}{\sqrt{2\pi\sigma_2^2}} \exp\left(-\frac{x^2}{2\sigma_2^2}\right)$$
(71)

Where \mathcal{E} , σ_1^2 , σ_2^2 are three parameters of this random variable law respectively called: contamination factor, first density variance, second density variance. A very small \mathcal{E} tends the Sum density to a Gaussian law; when \mathcal{E} higher than 0.3 tends the Sum density to asymmetric non Gaussian density. Below, some examples are given in order to understand the effect of these parameters on the Gaussian Sum density [1, 3, 7, 11, 15, 24, 25, 27, 31–32, 34, 41–44, 50, 61, 65, 69, 73, 76–78].

The choices for the contaminating pdf are various such as 'heavy-tailed' distributions, or the Laplacian, or the double exponential. But, most often, the ratio σ_1^2/σ_2^2 has generally the value between 1 and 10,000, in our estimation problems, this ratio has been varying between [50, 100, 200, 500, 1000, 2000]. This model has been privileged in order to simulate a scenarios with extreme denied GNSS environment. This approach has already been used to model non-Gaussian measurement channels in narrowband interference suppression, a problem of considerable engineering interest [57, 78].



FIGURE 9 Noise density ($\varepsilon = 0.5$) — Noise density ($\varepsilon = 0.85$)

When ($\varepsilon = 0.5$), it is easier to observe that in this simulation (see Fig.9), two Gaussian densities (blue and red) and Gaussian sum (green) are centered in addition to the curve variation in green colour which characterizes the impulsive contaminated noise. When the sum is not centered, it is possible to observe the sum of two Gaussian and the asymmetry of the non-Gaussian density easier than in the previous case ($\varepsilon = 0.85$). It is the best asymmetry in order test nonlinear filtering robustness.



FIGURE 10 Pdf of non-Gaussian density function of ($\varepsilon = 0.005$) for non-centered noises

Finally, for ($\varepsilon = 0.005$), one can assume that there is no impact on the final estimation based on Gaussian sum filtering, but as it is shown in the right figure, the impulsive noise is still varying from the Gaussian density and requires then to be tested also in our simulations.

Robust Gaussian Mixture-CKF

In this case we propose to use two parallel CKF to filter the density described in the previous paragraph and given by the eq.71. It is then possible to use two parallel CKF algorithms for each Sum pdf characterized by σ_1^2 , σ_2^2 . Two CKF algorithms based on eq.51 till eq.62 are applied simultaneously with replacing the innovation by the following equation eq.72.



Simulation 2. (Better RMSE with robust Filters)

FIGURE 11 North Velocity Mean Square Error — Down velocity Mean Square Error

$$S_{k/k-1} = \frac{1}{2n} \sum_{i=1}^{2n} Y_{i,k/k-1} Y_{i,k/k-1}^{T} - \hat{y}_{k/k-1} \hat{y}_{k/k-1}^{T} + \sigma_{j}^{2} \dots j = 1,2$$
(72)

And by additional estimation step described by the following equations :

$$\xi_j^2(k/k-1) = S_{k/k-1}$$
 and $\gamma_j(k) = N(e_j(k/k-1), \xi_j^2(k/k-1))...j = 1,2$ (73)

$$\alpha_{j}(k) = \frac{\lambda_{j}\gamma_{j}(k)}{\sum_{j=1}^{2}\lambda_{j}\gamma_{j}(k)}.....j = 1,2$$
(74)

$$\hat{x}_{R}(k) = \sum_{j=1}^{2} \alpha_{j}(k) \hat{x}_{j}(k/k)$$
(75)

$$\hat{P}_{R}(k) = \sum_{j=1}^{2} \alpha_{j}(k) \hat{P}_{j}(k/k) + (\hat{x}_{j}(k/k) - \hat{x}_{R}(k))(\hat{x}_{j}(k/k) - \hat{x}_{R}(k))^{T} \dots j = 1,2$$
(76)

With
$$\xi_j^2(k/k-1) = S_{k/k-1}$$
 and $e_j(k/k-1) = y_k - \hat{y}_{k/k-1}$.

It is then observed that the proposed robust Gaussian Mixture filters provide much better estimation with better accuracy comparing with other methods and applications founded in literature. It is possible to observe on direct estimation results, accuracy of EKF, DD1, DD2 with reformulated versions of these algorithms. The same phases such as for pitch estimation are observed and again, Robust Gaussian Mixture DD2 shows much better results than other non linear filters such as EKF, DD1, DD2, Gaussian Mixture EKF and Gaussian Mixture DD1. It is clear that the only filter able to track real velocity is GM-DD2. Again, for roll and down velocity, even with some instability, GM-DD2 and GM-EKF provide better estimation compared to GM-DD1[42].

Robust Adaptive Non Linear Filters

New results have been carried out in this section. which could be implemented in real time application in the presence of interferences especially, due to urban canyon, navigation in forest, low altitude flight,...etc. By those proposed hybrid techniques, both methods proposed previously are combined or hybridized. The resulted hybrid robust non linear algorithms are given by the factor described in eq.63–66 and the CFK algorithm.



FIGURE 12 Gaussian Mixture SPKF-CKF Design

The simulations are computed based on GPS, gyroscopes and accelerometers outputs, in order to validate the efficiency of the proposed architecture based on nonlinear filters 'EKF, SPKF and CKF'. Direct estimation for INS/GNSS model is compared, based on Gaussian Sum forms of these algorithms. Simulation of new integrated navigation design is compared with classic nonlinear filtering algorithms and ACRLB [1]. All these techniques are finally computed under outliers conditions of GNSS and additional noise effect in the system process. This additional disturbance serves to observe the effect on each category of filter. *Simulation conditions* N = 1000; dt = 0.005; g = 9.81m/s/s; $\varepsilon = 0.1$; $\sigma_2^2 = 2000 \sigma_1^2$. Below attitude and velocity estimation based on EKF, UKF, CKDF and CKF for Gaussian processes and non-Gaussian processes.

It is interesting to observe and compare the efficiency and the accuracy of each filter against non-Gaussian process noise. In simulation, only attitude angles and velocity estimations are presented because of nonlinearity of these state variables given in the state model.



FIGURE 14 Down Velocity MSE - Down Velocity MSE (Zoom)

It is possible to observe that for vertical velocity and pitch angle estimation (see Fig.13–14), CKF presents superior accuracy without disturbances. Velocity estimation is then more accurate with new formulated algorithms proposed in this work and UKF, CKF are better estimators than CDKF and EKF based on Gaussian mixture. It is then possible to observe that CKF outperforms UKF, CDKF and EKF comparing to ACRLB, see [Benzerrouk, 5].

2.6 Gaussian Mixture Adaptive CKF GM-ACKF

The third chapter is ended by simulation results of INS/GNSS under interferences modified as an impulsive noise with high error in the initial state.

In Fig.15, it is possible to observe in purple colour results of attitude, velocity estimation using hybrid robust proposed technique called Gaussian Mixture Adaptive SPKF-CKF-GHKF. Finally, the chapter contains significant results with three main contributions such as explained in the previous sections, first, adaptive fading non linear filters synthesis, Gaussian Mixture non linear filters derivation and hybrid robust non linear filter based on Gaussian Mixture Adaptive non linear filtering. In the next section, overview of the new developments contents is described, see [Benzerrouk et al., 2].



FIGURE 15 Pitch error estimation — East velocity estimation

2.7 Aerospace Sensors and Integrated Navigation Systems

This part is very important and provides actual overview of sensors development in Aerospace, such as accelerometers, gyroscopes, magnetometers, compass, baroaltimeters, radioaltimeters, laser telemeters,...etc, well described and compared, given a large view on the most modern technology used in building these sensors, with several application examples in Aerospace. First part of this chapter contains also Satellite navigation systems description including GPS, GLONASS, Galileo system components and frequencies spectrum. The second part of this chapter is dedicated to integrated navigation system based on the fusion of inertial sensors and satellite navigation systems described and analysed in a large specialised literature. The inertial navigation system is the kernel of all integrated navigation systems developed in this

dissertation. First, the problem of UAV's integrated navigation system INS/GNSS is simulated under several initial conditions:

a. APPLICATION TO Quadrotor "Return to Home" OPTION. Return Home Scenario of quadrotor UAV AR.DRONE 2.0 (see Fig.16) without GPS cannot be achieved successfully only with inertial sensors and even with visual aids, this function is not available and not activated onboard the navigation board of the quadrotor UAV only if GPS flight recorder is connected to the mother board. Thus, our interest in solving a new problem related with such option in GNSS denied environment, especially GPS which used on UAV or robot. The selected reference trajectory has been a closed triangle with distance variation between the three extremity points from 100m to 250 m and 500m, see Fig.16.



FIGURE 16 2D Trajectory control of AR.Drone 2.0 Quadrotor UAV using AR.Free Flight on Android

The state machine to control various flight modes of AR.Drone are described and analyzed in details.



FIGURE 17 North velocity estimation — Pitch angle MSE

COMMENTS: By the analysis of GNSS signal and state estimates based on EKF, UKF, CDKF, CFK on Fig.16–17 it is possible to observe the impulsive nature of the interfered signal or jammed signals with the influence on non linear filtering

algorithms, in most figures, represented in blue colors. Usual available jammers typically transmit chirp signal in which the frequency increases ('up-chirp') or decreases ('down-chirp') with time, such as for multiple RADAR and SONAR signal processing problems. The power of jammers is over 10 dBm. High-power jammers shall have a fatal impact on GNSS usage over a large navigation zone such as our flight experience in a park. It is clear on multiple state estimation results for position, velocity and attitude of the quadrotor UAV that the most efficient algorithm is the Gaussian Sum CKF designed on figures by "NGCKF" with purple color in GPS-denied environment during automatic flight trajectory and remote control of AR.Drone 2.0.

b. INS/GNSS/COMPASS/Lidar for autonomous robot navigation. As described in previous chapter sections, different fusion architectures existent and have been widely investigated in specialized literature. In this dissertation, the main problem which was solved is the "robust integration algorithm of sensors data fusion". For multiple sensors fusion, information filter algorithm based on CKF and GHKF have been selected according the accuracy and have been applied to robot integrated navigation system and UAV's sensors fusion. In denied GNSS environment, Fig.12 shows the proposed solution based on parallel CKF or Cubature Information Filter CIF for robust estimation of navigation states.

c. Non linear dynamic system of robot: kinematic model is used for this experimentation instead of its dynamic model because the main problem processed in this dissertation is the state estimation of sensor fusion and not the control of the robot.



FIGURE 18 2D robot model (Robot navigation)

This model can be extended to consider other parameters such as wheel radius and slip angle that can have significant importance in other applications such as described on Fig.18. Also, GPS, GLONASS and electronic compass are assuming delivering data at 10 Hz, 5Hz and 5Hz respectively.

$$\begin{bmatrix} \dot{x}_{L} \\ \dot{y}_{L} \\ \dot{\phi}_{L} \end{bmatrix} = \begin{bmatrix} v_{c} \cdot \cos\left(\phi\right) - \frac{v_{c}}{L} \cdot (a \cdot \sin\left(\phi\right) + b \cdot \cos\left(\phi\right)) \cdot \tan\left(\alpha\right) \\ v_{c} \cdot \sin\left(\phi\right) - \frac{v_{c}}{L} \cdot (a \cdot \cos\left(\phi\right) + b \cdot \sin\left(\phi\right)) \cdot \tan\left(\alpha\right) \\ \frac{v_{c}}{L} \cdot \tan\left(\alpha\right) \end{bmatrix}$$
(77)

The velocity is generated with an encoder located in the back left wheel. Due to multiple sensors integrated in this application, and due to the previous results issued after synthesis of non linear high order information filters, Cubature Information Filter CIF is proposed to solve the problem of Robot navigation in Urban environment. Below simulation of robot navigation based on sensor fusion IMU/GPS/COMPASS and encoder with Gaussian assumption of measurement and process noises.

COMMENTS: This application is very important in aerospace especially for actual and future Mars and Moon exploration missions using robots and rovers [74]. The problem of measurement outliers is realistic and is considered for multiple space missions based on Data fusion.



FIGURE 19 East Distance-2D Vehicle model (Robot navigation)

1. After multiple simulations, CIF works well and outperforms UKF and CKF on the basis of RMSE criteria and multiple sensor fusion capabilities with less computational time complexity, thus, it is proposed to apply the decentralized Cubature Information Filter based on non Gaussian measurement noise. The algorithm described in the diagram in fig.62 is then applied. It is shown after analysis of different figures and results that the developed robust algorithms work well in such denied GNSS environment with significant and relevant improvement of the existing system has been achieved.

c. Blind Peoples Localization and Navigation System. Patent N° 89221,issued on 08 November 2009, Moscow, Russia. The principle is described in the following figure:

Tests are computed based on gyroscopes and accelerometers outputs during walking with IMU mounted on foot in order to validate the efficiency of the proposed architecture based on non linear filters. EKF, CKF and CIF.

d. EXPERIMENTAL TESTS. A real time application of the shoe-mounted inertial navigation system was tested [openshoe]. The system hardware is composed of: -a laptop PC-IMU **Analog Device 16367-250 Hz** frequency of accelerometers, gyroscopes and magnetometers — Stance-Still step detector and an assisted AUPT-ZUPT based on "**Direct Cubature Kalman filtering approach**".

2nd approach for Pedestrian navigation system based on GNSS receiver:

On the basis of previous results using CKF and CIF filtering algorithms, it is proposed then to apply the Gaussian Sum based CIF to the problem of Pedestrian navigation using a different nonlinear model instead of Foot-mounted IMU's, the use of GPS/GLONASS receiver such as integrated on GALAXY Smartphone Tab3. The backup positioning system during GNSS missing is assumed to be GSM network based on base stations [61–62].

Novel Gaussian Sum Information Filter

Robust Gaussian Sum-Cubature Information Filter. This section describes the CIF algorithm, which uses CKF in an EIF framework. The CIF algorithm is summarized. The factorization of the error covariance matrix, evaluation of cubature points and propagated cubature points for the process model is required for CIF. Let the information state vector and information matrix be given by b yk|k-1 and Yk|k-1. The factorization of the inverse information matrix is required to evaluate S, which is required for cubature points propagation.

$$[Y_{k-1/k-1}]^{-1} = S_{k-1/k-1}S_{k-1/k-1}^{T}$$

In the measurement update of CIF, the first two steps involve the evaluation of propagated cubature points and the predicted measurement.

The information state contribution and its associated information are explicit functions of the linearized Jacobian of the measurement model. But the CKF algorithm does not require the Jacobians for measurement update and hence it cannot be directly used in the EIF framework. However, it is possible to rewrite the CKF update in the EIF framework. The predicted measurement can be given by the following approximations:

$$P_{zz,k/k-1} \approx \nabla h_x P_{k/k-1} \nabla h_x^T$$
(79)

Then, we obtain the following expression :

$$I_{k} = P_{k/k-1}^{-1} P_{xz,k/k-1} R_{k}^{-1} P_{xz,k/k-1}^{T} P_{k/k-1}^{-T}$$
(80)

With

$$i_{k} = P_{k/k-1}^{-1} P_{xz,k/k-1} R_{k}^{-1} \Big[\nu_{k} + P_{xz,k/k-1}^{T} P_{k/k-1}^{-T} \hat{x}_{k/k-1} \Big]$$
(81)

$$P_{xz,k/k-1} = \frac{1}{2n} \sum_{i=1}^{2n} X_{i,k/k-1} Z_{i,k/k-1}^{T} - \hat{x}_{k/k-1} \hat{z}_{k/k-1}^{T}$$
(82)

The updated information state vector and information matrix for the CIF can be obtained by using Ik and ik from (eq.80) and (eq.81).



FIGURE 20 Gaussian Sum Based Cubature Information Filter applied to Pedestrian Navigation State estimation Problem

Thus, the Gaussian sum based CIF can be derived and applied to the model of pedestrian navigation system based on multiple sensors fusion. On the basis of the kinematic model given in the previous section and during multiple simulations in denied GNSS environment with Gaussian mixture noise, the following results have been carried out and observed on Fig.20. It is possible to observe that the Gaussian Sum based CIF algorithm is the best estimator able to track the true trajectory in the

North-East frame, in addition to different state estimation of the azimuth, the north distance and east distance. The Gaussian sum information filtering is more accurate and represents an alternative to all previous developed approaches.

IRIDIUM SATELLITE BASED POSITIONING SYSTEM

Iridium service provides real Telecommunication capabilities to users worldwide. Continuous global coverage is realized using 66 space vehicles (SVs), distributed among 6 planes in near circular orbits at 86.4deg inclination, orbiting at an altitude of 780km (much lower than the 20,000km GPS orbit altitude). A 31.6deg angle separates each co-rotating orbital plane, and the remaining 22deg angle separates the two planes at the seam of the constellation, where spacecraft are counter-rotating. Iridium spacecraft spend on average 10min in view of a given location on the surface of the earth, and circle the earth in a period *IRI T* of 100min 28s [9].



FIGURE 21 Iridium Satellite constellation around the earth (66 satellites)

2. Short burst data — SBD geodata

- The first tests have been observed during a week during all day in static mode in Algeria.
- Then, a second test has been achieved in another Geographic area in Europe from Belgium, France to Spain in Dynamic mode, when a real tracking based on Iridium measurements has been realized.



Real results of IRIDIUM only based Positioning during GPS missing

FIGURE 22 Static Iridium Localization during GPS missing-Circle center initialization-Latitude-Longitude Plan

STATIC TEST RESULTS: Iridium positions converge to the center of the circular zone which approximately 36m-467m the GPS location of the Tracker. The problem of increasing accuracy in the static mode could be solved by geometrical solution which leads with circle fitting algorithms on the basis of multiple measurements of Iridium location estimation. The accuracy achieved 36 meters instead of 1km (see. Fig.22).

Method used in solving the initialization process of the circle center estimation:

We consider the following 2D problem: given a set of n points Pi(xi;yi) (i = 1: : :n) find the center C(xc;yc) and the radius r of the circle that pass closest to all the points. The underlying principles of the proposed fitting method are to first compute an initial guess by averaging all the circles that can be built using all triplets of non- aligned points, and then to iteratively reduce the distance between the circle and the complete set of points using a minimization method. This induces the use of Gaussian Sum filters, each filter with its initialization. **This solution is under Patent consideration for real industrial use.**

Let Pi(xi;yi), Pj(xj;yj) and Pk(xk;yk) be three points. Elaborate algorithms are used and applied, main important are: Iterative improvement, LEVENBERG-MARQUARDT method which is a combination of the GAUSS-NEWTON and steepest descent methods (see Fig.22) [under patent]. Robust estimation approach such as given in [14, 18, 19] are also investigated.



FIGURE 23 Dynamic Iridium Localization during GPS missing (Iridium Outliers-Impuslive noise)

In static mode, the initialization is fixed to 10 minutes, receiving Iridium Short Burst data every 10 sec, which provides 60 measurements. On the basis of previous specialized researches, Gaussian sum filtering is the applied based on Cubature Kalman Filter Algorithm achieving accuracy equivalent to 36 meters.

DYNAMIC TEST RESULTS: Iridium positions are delivered every 10sec with an average error of 1km during a very long trajectory from Belgium through France and to Spain. It is observed phenomena of outliers varying from 2km to 10km and in seven (07) cases superior than 100 km (high outliers). Those outliers are considered as impulsive measurement noises and in this case a Gaussian sum based nonlinear filters are proposed as an alternative to the problem discovered during multiple tests (see Fig.23).

$$VR_{j}^{i} = \rho_{j}^{i} + w_{k} = \sqrt{\left(x_{j} - x^{i}\right)^{2} + \left(y_{j} - y^{i}\right)^{2}} + w_{k}$$
(83)

It is a Multi-positioning system in denied GNSS environment. There is no Gaussian assumption on the measurement noise w_k . The idea is to use the same kinematic model of 2D navigation and consider each iridium location measurement as an artificial beacon or virtual beacon. The measurement covariance matrix is time varying and is dependent of the C.E.P radius of each incoming iridium Geo Location data (Under Patent).

2.8 General conclusion

Novel forms of Gaussian mixture formulation based on EKF, SPKF, GHKF and CKF provide much better results and excellent convergence of estimation errors for position, velocity and attitude angle estimation in non Gaussian measurement

environment. These algorithms are recommended in presence of impulsive noises. Robust adaptive fading non Gaussian filters are a good solution in presence of interference at initialization step and during navigation. These algorithms are called "Robust" or Gaussian mixture Adaptive Non Linear Filters. Accurate estimation was resulted and much efficient integrated navigation system INS/GNSS has been developed. Decentralized design based on CIF and Gaussian Mixture synthesis is proposed as a fusion algorithm under severe conditions. Several applications are potential host for the proposed robust designs, such as: robot navigation, Ship navigation, aircraft navigation, UAV, pedestrian navigation, Iridium Satellites based positioning ...etc.

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