## Review

## of the dissertation "Modern approach in nonlinear filtering theory applied to original problems of aerospace integrated navigation systems with non-Gaussian noises" of Hamza M. Benzerrouk

The dissertation contains deep investigation of the nonlinear filtering problem and application of the developed algorithms to various navigation systems. Nonlinear models describe observation processes in the unmanned aerial vehicles (UAV), satellite communication and tracking systems, robot motion and manipulators. Unlike linear models, the optimal estimation problem for nonlinear systems is proved to have no finite dimensional solution. That is why any new approach gives benefits in some specific areas of application but a new research is still needed for other cases.

Thus, the nonlinear dynamic estimation problem has important application and the mathematical theory is not completed. The author studies new classes of the nonlinear observation systems and presents results of the proposed algorithms.

The main content of the dissertation can be divided into three parts: problem statement and description of the standard and new algorithms, simulation and competitive analysis, application to real systems.

The full nonlinear estimation system of the dissertation is presented in Fig. 1. All particular algorithms are described in section 2. One of the main applications concerns UAV. The corresponding mechanical system is shown in Fig. 3 where the standard four systems of coordinates are defined. Some of them can be reduced to the simplified system in (4)-(11). Various sensor disturbances are noted for the satellite observation model.

In section 2.2 the problem is formulated in the general form. The extended Kalman filter is given in the two forms: in the standard recursive form and in the information terms. The first essentially nonlinear filter appears in the next subsection, and it is the central difference Kalman filter (CDKF). The idea of the algorithm is correctly explained. It consists of an increase of the covariance matrix by a term containing convexity features of the nonlinear observation function.

The same idea is then implemented for the multidimensional interpolation using quadrature formulae. In section 2.3 the Gauss-Hermite Kalman filters are described in detail.

In section 2.4 the author presented adaptive fading procedure that modifies the previous approach. The reason for modification is not explained but the simulation results show an advantage of the new algorithm.

A specific case of Gaussian mixture of noises is studied in section 2.5. A new algorithm is proposed that differs from the conventional Huber robust approach.

Finally, a number of applications are described. All estimation algorithms from previous sections are applied to nonlinear estimation problems. The presented curves show a good performance of the new algorithms.

The paper contains some drawbacks. The main problem for the reader is to understand what is shown in the formulae because variables are sometimes not defined or explained.

1. An evolution operator for time varying linear system is not equal to exponent of the integral of the system matrix, as it is written in (4). In addition, two mistakes present in

the second order approximation in (4): the linear term as to be added, and denominator in the second order term must be squared.

- 2. The algorithm (24)-(41) is not described in detail. A reader has to presuppose that  $\bar{\nu}$  is the unknown mean value of the noise  $\nu$ , and  $f(\chi^x_t, \chi^\nu_t, u_t)$  is the corresponding augmented state evolution function. But the values of  $\chi^x_{i,k/t}$  and the index i are not explained in this section. The choice of coefficients in (41) is not proved.
- 3. A suboptimal fading factor  $\lambda_{i,k}$  is introduced in section 2.4. It seems this factor does not depend on i. The vectors of  $\nu_k$  are not defined in the equations (63). The reason of this new procedure is also not explained. Proof is replaced by simulation results in Fig. 7 and 8. But the data and the problem statement in those experiments are not given.
- 4. A Gaussian mixture for the noise distribution is studied in section 2.5. The proposed procedure of estimation in (72)-(76) is very similar to the Bayes' rule. The Bayesian approaches are very popular in nonlinear filtering, but no comparison and references are presented.
- 5. All algorithms of nonlinear filtering are illustrated with experimental results in figures. But the problem statements of the experiments are not described in detail. A dimension of the model, a nonlinearity, a disturbance covariance and a source of the data are not presented in the paper. It is difficult to verify a quality of the estimation algorithms by curves of an unknown experiment.
- 6. There are no theorems in the dissertation. The proposed algorithms are interesting but conditions for perfect implementation to other applications cannot be obtained.

Nevertheless, I guess the paper satisfies the conditions for the PhD dissertations in mathematics of the Saint Petersburg State University.

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